Grade 10 Unit 4 - Electromagnetic Induction Notes

Aaron G. Kebede

May, 2022

Dynamo Effect and Electromagnetic Induction

We have seen a lot of electromagnetic effects in the past. To recap, we have seen Ampere's Law, Motor Effect, and Hall Effect. What all these effects have in common is the generation of magnetic field from current. What we are about to see is the reverse process- the generation of current from magnetic field. To get started, let's learn a few important, but trivial topics:

B - we have seen this physical quantity as being the *magnetic field strength*. This same **B** is called the *magnetic flux density*. Mathematically,

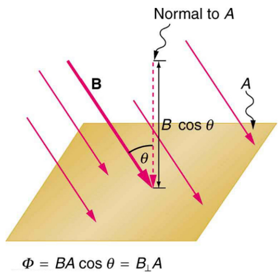
$$B = \frac{flux}{A}$$

We denote flux(the magnetic field lines that cross a given area) by the Greek later phi(ϕ).

$$B = \frac{\phi}{A} \iff \phi = BA$$

Thus, we can see that the SI unit of this so-called flux is Tm^2 and it is called *Weber(**Wb**)*. However, flux is maximum when it penetrates through the area perpendicularly. To understand, let's define a vector.

Area Vector(**A**) - is a vector of a planar surface whose magnitude is equal to the area of the surface and its direction is perpendicular to the surface. In this case, for our flux to be maximum, our area vector should be parallel to the magnetic field lines. Thus, we have the following be true.



In that case, our magnetic field is given by:

$\phi = BAcos\theta$, where θ is the angle between the are vector and magnetic field lines.

When Michael Faraday was doing experiments involving induction of current from magnetic he noticed one important thing: voltage always arose when the flux was changing - not necessarily the magnetic field. Let's see ways how we can change the flux:

$$\phi = BAcos\theta$$

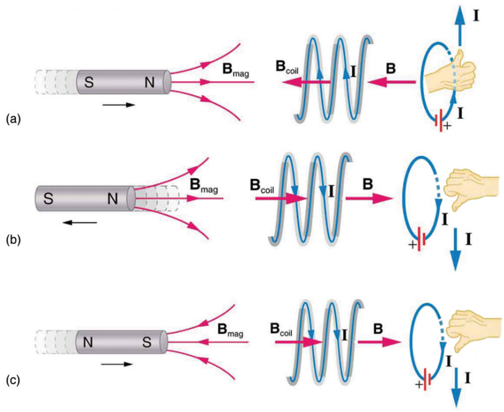
We can change the flux by changing the magnetic field, the area or the orientation of the system. Let's consider a a loop of a conductor in a magnetic field, the easiest out of all the three quantities to change is obviously the angle. We can rotate the loop and each time, we can keep changing the orientation at each instant and hence have the flux changing to generate EMF. Think of hydroelectric power plants for instance. You have heard probably heard about gathering a lot of water in the dams and releasing it at a pace to rotate turbines which in turn generate electricity. This rotation of turbines causes changes in the flux which in turn causes EMF to generate. This is Faraday's law in a nutshell.

Faraday's Law states that the magnitude of the induced EMF is proportional to the time rate of the change in flux. Mathematically,

$$\varepsilon \propto \frac{\Delta\phi}{\Delta t}$$

Faraday's law tells us about the magnitude of the induced EMF. However, the direction of the induced EMF is given by a law called Lenz's Law.

Lenz's law tells us that *the direction of the induced current is such as to oppose the change causing it.* To understand this law, let's consider the following situations below:



Let's discuss the first case. In **(a)**, the north pole of a magnet is approaching the coil of wire and since the flux is changing, EMF is being induced. We can deduce the magnitude of the EMF being induced using Faraday's law. However, to determine the direction of the induced current, we use Lenz's law. Since the north pole of the magnet is approaching the coil, Lenz's law tells us that the direction the induced current acts so as to oppose the change causing it(the change is the North pole approaching the coil). Thus, the induced current will create a magnetic field that would oppose the applied magnetic field. In that case, the magnetic field by the induced current is going to be a North pole on the left side of the loop to oppose the motion of the magnet causing the induction. Then, we can use the Right Hand Rule to see the direction of the current that causes such a magnetic field.

In case **(b)**, the north pole of the magnet is moving away from the coil of wire, thus the to oppose this change, the magnetic field by the induced current at the end of the coil by the magnet should be the south pole. Thus, we can use the Right Hand Rule to determine which direction of the current will induce such an orientation of magnetic field on the solenoid.

Thus, we can generalize the dynamo effect in the following equation(considering there are N loops of wire getting crossed by the magnetic field):

$$\varepsilon = -\mathrm{N}\frac{\Delta\phi}{\Delta t}$$

The negative sign addition we have here is to signify that the EMF generated will have an opposing effect to the change causing the induction.

Self and Mutual Inductance

Self-inductance

When current is changing through a conductor, we can see the effect of Faraday's law on it. We know that when current is flowing through a conductor, there is a magnetic field. And when the current changes, there will be a change in the magnetic field and as a result, there will be a change in magnetic flux and hence, current will be induced. This phenomenon is called **self-inductance** and we can explain it using Lenz's law. The induced EMF is proportional to the time rate of change in the current:

$$\varepsilon \propto \frac{\Delta I}{\Delta t}$$

The constant of proportionality for the above relationship is called self-inductance(**L**), thus we have the following:

$$\varepsilon = -\mathrm{L}\frac{\Delta I}{\Delta t}$$

- The negative sign on here is also a result of Lenz's law. It tells us that the EMF that is induced opposes the change in current.
- The SI unit of self-inductance is Henry(H). *Do you remember that the SI unit of permeability was* H/m?

Self-inductance implies opposition to change in current, this implies that for a conductor with a large self-inductance, it is hard to achieve change in current quickly.

Self-inductance of a solenoid

We have seen that self-inductance is given by:

$$\varepsilon = -\mathrm{L}\frac{\Delta I}{\Delta t}$$

We also know that the EMF given by the dynamo effect is given by:

$$\varepsilon = -\mathrm{N}\frac{\Delta \phi}{\Delta t}$$

When we combine the above two equations together, we get,

$$-\mathrm{N}\frac{\Delta \phi}{\Delta t} = -\mathrm{L}\frac{\Delta I}{\Delta t}$$

We see that the self-inductance(L) is given by:

$$L = N\frac{\Delta \phi}{\Delta I}$$

We know that for a solenoid, the magnetic flux density is given by:

$$\mathbf{B} = \mu_0 n I$$

$$L = N\frac{\Delta BA}{\Delta t} = N\frac{\Delta(\mu_0 n I)A}{\Delta I}$$

Since $n = \frac{N}{L}$, we have:

$$\mathrm{L} = \frac{\mu_0 N^2 A}{L}$$

Mutual-inductance

Mutual inductance, similar to self-inductance is the generation of current on another device due to the change in current in one device. It is the effect of Faraday's law. As the current in one device changes, the magnetic fluxes due to the current changes and hence nearby conductors will experience a flux change through them and that will induce EMF. We can determine the direction of the induced current using Lenz's law and the magnitude of the induced EMF is proportional to the change in current.

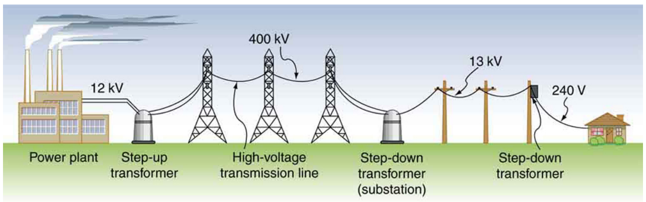
$$\varepsilon \propto \frac{\Delta I}{\Delta t}$$

$$\varepsilon = -\mathrm{M}\frac{\Delta I}{\Delta t}$$

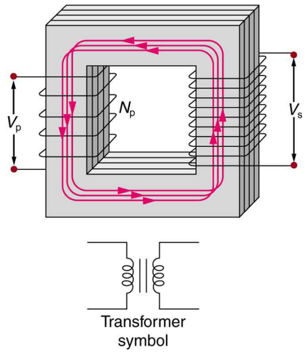
M is called the mutual-inductance of a given conductor and it is the measure of the effectiveness of inductance of current.

Transformers

Transformers work using the concept of mutual inductance. As the name indicates, they transform voltage from one value to another. Power is sent long distances at high voltages, because less current is required for a given amount of power. But high voltages pose greater hazards, so transformers are employed to produce lower voltage at the user's location. Based on the amount of the output voltage we categorize transformers into step-up and step-down which amplify and decrease the voltage respectively.



Let's consider the transformer below, for instance:



For a simple transformer such as the one shown above, the output voltage depends almost entirely on the input voltage and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage to be:

$$V_s = -\mathrm{N_s}\frac{\Delta\phi}{\Delta t}$$

Since the rate of change in flux is the same for each case, we have the following equation(the **Transformer Equation**) be true:

$$\frac{V_s}{N_s} = \frac{V_p}{N_p} \iff \frac{V_s}{V_p} = \frac{N_s}{N_p}$$

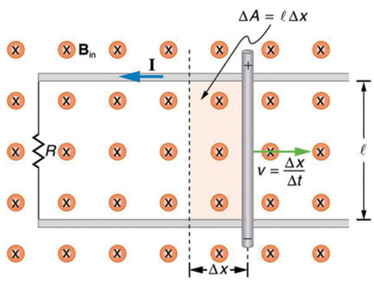
Since power is conserved, the input power(primary) of the transformer should equal the output(secondary) in ideal cases.

$$P_p = P_s$$
$$V_pI_p = V_sI_s$$
$$\frac{V_p}{I_s} = \frac{V_s}{I_p}$$

Thus, we can see that whenever we have a step-up transformer, the current is bound to decrease while in a step-down transformer, the current increases.

Motional EMF

Whenever flux is changing through a conductor, EMF is induced. We have seen when discussing Dynamo Effect that flux change occurs as a result of relative motion between the source of magnetic field and the conductor. In this specific case, we will look at the case in which the conductor is moving.



We have seen one case of motional EMF early on in this unit when discussing the Hall Effect. Consider a conductor moving through a magnetic field as shown in the figure above. As the conductor is moving through the field, EMF is induced according to Faraday's law:

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

$$\varepsilon = -N\frac{\Delta BA}{\Delta t}$$

$$\varepsilon = -N\frac{Bl\Delta x}{\Delta t}, \text{ but } \frac{\Delta x}{\Delta t} = v$$

$$\varepsilon = -N\frac{Bl\Delta x}{\Delta t}$$

$$\varepsilon = -NBlv$$

In our case, since we only have one conductor, N=1. Thus,

$$\varepsilon = -Blv$$

However, we see that the induced EMF is perpendicular to both the velocity and the magnetic field.