Grade 10 Physics Notes - Part 2 on Unit 2

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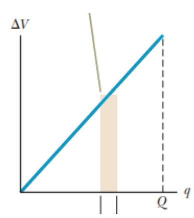
Energy Stored in a capacitor

We have discussed previously that the use of capacitors is to store more charge at a lower potential(compared to other electrical equipments such as batteries.) When charges are stored on the plates of a capacitor, there is a potential energy that is stored in the capacitor as the plates are charged and are kept at a distance(think of the plates as point charges and a distance **r** between them).

To find the energy stored in the capacitor, let's see how a capacitor's charge increases with respect to the voltage.



We can see on the graph above that when a capacitor charges, the potential difference increases *proportionally* with the charge being stored. When we have such situations, we take the average value of the charge. Since the charge is increasing with a constant slope, we take the midpoint of the line.

That is,

$$\frac{Q+0}{2}$$

That gives us the average value of the charge while the potential difference on the plates of the capacitor reach the potential difference of the battery. If we look at the area under the curve of the **V** vs **Q** curve, we get the following:

$$\text{Area} = \frac{1}{2}Q \times V$$

The area under the line happens to be the energy stored in the capacitor. Thus,

$$E = \frac{1}{2} Q \times V$$

We can also express the area in terms of the capacitance and voltage.

We know that

$$Q = C \times V$$

That means,

$$E = \frac{1}{2}(C \times V) \times V$$

$$E = \frac{1}{2} C V^2$$

To understand the mathematical meaning of the $\frac{1}{2}$ at the beginning of the equation, we can make use of calculus and interested students can look at the following.(Remember how we used calculus to estimate the amount of charge left in a capacitor during discharge?)

$$dW = \Delta V \times dq$$

$$dW = \frac{q}{C} \times dq$$

To get the work, we integrate both sides:

$$\int dW = \int_0^Q \frac{q}{C} dq$$

$$W = \frac{q^2}{2C}\Big|_0^Q$$

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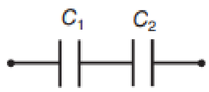
$$W = \frac{Q^2}{2C}$$

Now, we can see how the $\frac{1}{2}$ has come to presence mathematically.

Combination of Resistors

More often than not, we will use multiple capacitors in a circuit than just one. Thus, we have to compute the effective capacitance of the circuit.
We can combine the capacitors through a single path end-to-end with the same charges passing through them. This connection is called a **series** combination of capacitors. Look at the figure below for an example of a series combination of capacitors.



In both capacitors, we have the same charges flowing through them. Let the charge provided by the battery be **Q**, and if these capacitors are connected with series to the battery, a charge of **Q** passed through each of them. But, the potential difference provided by the battery is divided between the two capacitors. If the potential difference provided by the battery is **V**, we have the following:

$$V = V_1 + V_2$$

We know that $V = \frac{Q}{C}$, thus,

$$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Since all the charges are equal,

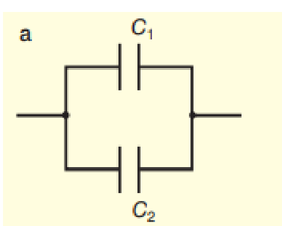
$$Q_1 = Q_2 = Q$$

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Thus, the reciprocal of the effective capacitance of the circuit is the reciprocal sum of each capacitance. Here, we can see that the effective capacitance can never be larger than the capacitance of each capacitance.

In situations where the capacitors are connected in parallel(when the wire **branches** out), as in the figure below,



We have the potential difference to be the same for the capacitors connected in parallel. However, the charge from the battery is the sum of the charge in each capacitor.

$$V = V_1 = V_2$$

But we know that:

$$Q = CV$$

And

$$Q = Q_1 + Q_2$$

$$CV = C_1V_1 + C_2V_2$$

$$CV = V(C_1 + C_2)$$

$$C = C_1 + C_2$$

This means that the effective capacitance of capacitors connected in parallel. is the sum of each capacitance.