Linear momentum and impulse

**Why we concern for linear momentum**

From Newton third law we can see that when two objects collides irrespective of their mass they experience equal and opposite force but we never know what will happen after the impact. They may head in the same or opposite direction, or may also stop. However we don’t know which one is happen. We don’t know the direction and magnitude of velocity after the collision. Since it’s difficult to precisely measure the magnitude of the force which appears during short time of collision.

However for such case the velocity can be easily solved using the concept of conservation of linear momentum without being worried on the force exerted (which leads to acceleration and velocity). Having good understanding on momentum may help you to have a point pool game since you may make a good guess where the ball is going to head after strike by the ball you hit(be safe Don’t try this), to determine the velocity space craft , etc. We will come on this a bit later. But let’s start with the concept momentum itself

**What’s linear momentum?**

Linear momentum is defined as the product of mass and velocity. It has a unit of kgm/s . It’s a vector physical quantity with the direction towards the velocity of an object. Quantitatively momentum tells you the amount of the effort needed to stop a moving object in given interval of time. Or its equivalent with the amount of effort needed to let an object move to certain velocity from state of rest. Higher momentum implies large force is needed to stop a moving object for given set of time. Mathematically momentum is given as

Example

A 3.00-kg particle has a velocity of determine a) the x and y component of momentum b) the magnitude and direction of momentum

**Solution**

 (fourth quadrant)

Unlike to inertia momentum depends on both mass and velocity and it show the resistance not to stop rather than not to change its state of motion. For a given object the momentum is higher as velocity gets large but the inertia is the same. Momentum is also different from KE considering vector nature, concept even formula too

Hence two object with the same KE does not mean they have the same momentum

**Concept based problem**

If two objects have equal kinetic energies. How do the magnitudes of their momentum compare? If a) m1>m2, b) v2>v1

 then

**Concept based problem**

Your physical education teacher throws a baseball to you at a certain speed and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices: You can have the medicine ball thrown with (a) the same speed as the baseball, (b) the same momentum, or (c) the same kinetic energy. Rank these choices from easiest to hardest to catch.

**Concept based problem**

In the absence external force bullet and gun have identical momentum before and after firing , if that is so why is difficult to stop a bullet than the gun

Solution

It’s obvious that the momentum of the bullet is equal with the momentum of gun but they do have different KE . For given momentum the higher KE is the one difficult to stop, since to stop a body its total KE must be absorbed or transformed. Now let’s compare the two KE

 in magnitude

Since t then

This makes hard to stop since large amount of energy needs to absorbed

**Impulse, J**

For the momentum to change either the mass or the velocity has to be changed. If we keep the mass to constant then the velocity has to be change for the momentum to be changed. Then from Newton second law, the net force must be applied to change the velocity. Why did you think you wear seat belt. The change in momentum is called impulse. Denoted by J and has a unit of kgm/s or Ns

From Newton second law of motion

Hence **Impulse, J** can be defined as force that is applied in a given interval of time contact. From the above the amount of force required to change the momentum is high if the time it took to change if very small.

**Think ? why do recent car uses air bag system ?**

Remember that to stop a body having large momentum then we have two option one applying a huge force in short time or applying reasonable force in long time. In the case of air bag system, it helps us to stop taking longer time since the air bag deforms it took longer time. Without an air bag we may collide with the steering wheel, hence the steering wheel hard it took lower time to stop us hence we experience larger force and we get injured. Why did you think boxer needs to wear hand gloves? Why you feel pain while hitting the wall but not hitting mattress? Why did car needs crushing zone ?(please look your text book) for these

**Example (based from text book)**

You throw ball mass 0.4kg against a brick wall. Before the strike the velocity of the ball was with the speed of 30m/s due at 37 SW. After strike the wall, a ball movies with 24m/s due 30 SE. If the contact time 0.25sec then find

1. Momentum before and after collision
2. The impulse
3. The average force applied during this time

Answer

**Example** (**based from text book with slight modification**)

A boy drops a stone of mass 300 g from a height of 9 m. If the stone rebounds to one third of the initial height

1. What is the momentum of the stone just before and after the collision ?
2. What is the impulse of the stone and force applied if the time of contact is 0.05sec?

**Answer**

Let’s find the velocity just before it hit the ground

 (down )

Now let’s find the velocity just after the collision

 (up)

**Law of conservation of linear momentum**

Previously we have seen that , from these one can see if the resultant external force act on system the zero then the momentum of system remain unchanged . In this regard , law of conservation of momentum states that ***if the net external force on system is zero or if the system is isolated (free from external force) , then momentum system is remain constant***.

 Mathematically

If

For system having two interacting bodies law of conservation of linear momentum can be stated as**, *if two bodies collide or push each other apart and no net external forces beside one push the other, the total momentum of the two bodies remain unchanged.*** Hence the total momentum just before the collision or the interaction is equal to the total momentum after the collision or the interaction

Hence for the isolated system involving collision of two bodies (we will come the proof a bit later)

Note that the law of conservation of linear momentum holds true for only

* **Isolated system**, system where there **no external force** beside the force appears while the two bodies interact. Example An Astrought pushing one another in space with zero gravity .In actual case this difficult to attain
* **System with zero net external force**: In this system there are external forces beside the interacting force but the total or net force is zero or near to it. Example when two billiard ball (pool ball) collide in pool table there is weight, normal force and slight friction too .Here the weight is balanced by normal force and friction is small hence the resultant force can be considered as zero. Hence possible to apply the law

**Concept based example**

In which case does are you going to apply law of conservation of linear momentum justify your answer

1. The collision of two billiard ball in pool table
2. A man with the skate board pushing a heavy table
3. Two person with skate board pushing one another while standing with the skate board
4. The collision of a ball with a ground
5. The car accidentally collides with the wall
6. The collision of air molecules in your room
7. A man firing a gun while standing firmly on the ground
8. Collision in Newton cradles

**Proof for law of conservation of linear momentum**

The proof can be done considering two interacting bodies where the net external force is zero , consider two billiard ball in pool table. If the balls named to be A and B then

From Newton third law

**How do I solve problems involving conservation of linear momentum?**

* **Step1** check if it’s possible to apply law of conservation of linear momentum
* **Step2** Draw a sketch showing the system both before and after the collision or interaction. Include coordinate axes and label the initial and final velocity vectors
* **Step 3** Apply the conservation of momentum equation along the x and y direction , take the direction of velocity into consideration
* **Step 4** Find the unknown

**Example [left for you]**

Particle A is travelling at a velocity of [-2,6] m/s. It collides with particle B which has a velocity of [10,12] m/s. The particles move together. The mass of particle A is 2 kg and the mass of particle B is 3 kg. Find the velocity of the combined particles after the collision.

 at

Example [left for you]

In the previous example if the particle A moves with the velocity of 12m/s at 37Nw and collides with particle which was moving at the speed of 10m/s heading 40NE.After collision if the first particle moves at 10m/s at 10NW find the velocity of the second particle.

**Example [Left for you]**

A bullet of mass 0.01 kg is fired with a velocity of [20,0] m/s into a sack of sand of mass 9.99 kg which is swinging from a rope. At the moment the just before bullet hit the sack, the sack has a velocity of [0,0.2] m/s. Work out the velocity of the bullet and sack just after the bullet hits the sack , assume the bullet is embed in to the sac.

Example

A bullet of mass m1 is fired into a wooden block of mass m2 suspended on some light string. The bullets embedded into the block and the entire system swings through a height **h**. what is the initial velocity of the string expressed in terms of h, m1 and m2

Solution

Using conservation momentum

Now let’s find the common velocity after collision

Using the conservation of mechanical energy

Solving the above

Now lets come to the question,

**Example (from national exam with slight modification)**

Two spherical masses A and B of mass m and 4m respectively, hang from ceiling of strings L. Sphere A is drawn side to height ho and released as shown in figure below. The sphere A collides with sphere B; they stick together and swing a maximum height of h . What should be the height of h?

m0

 4m0

Example

Two identical objects with the same initial speed collide and stick together. If the combined object moves with half of the initial speed of either object, what was the angle between the initial velocities? (Assume one of the objects moves along east)

**Types of collision**

As it has been said before collision refers an event during which two particles come close to each other and interact by means of forces. The interaction forces are assumed to be much greater than any external forces present; hence during this case the total momentum is conserved irrespective of the type and complexity force. Hence as far as the system is isolated or net external force is zero the momentum of the system is conserved.

In contrast, the total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. Based on this collisions are categorized as being either elastic or inelastic depending on whether or not kinetic energy is conserved.

An **elastic collision** between two objects is one in which the total kinetic energy (as well as total momentum) of the system is conserved or remain unchanged. However in actual case there always be lost in KE due to sound, heat and deformation. However in if the energy lost by these is small then we can consider as elastic collision.

Example collisions between certain objects in the macroscopic world, such as billiard balls, collision of air molecules with one another are only *approximately* elastic because some deformation and loss of kinetic energy take place. For example, you can hear a billiard ball collision, so you know that some of the energy is being transferred away from the system by sound. An elastic collision must be perfectly silent! *Truly* elastic collisions occur between atomic and subatomic particles. Furthermore, there must be no transformation of kinetic energy into other types of energy within the system. But if the energy transformed to other form beside the KE to be small then we consider as elastic collision.

**Equations**

Since the KE is conserved we do have another equation

**For one dimensional elastic collision**

Rearranging momentum equation

 Equation 1

Rearranging energy equation

 Equation 2

Combining equation 1 and 2

***For two dimensional collisions the above equation are not valid since it does not account the vector nature of the velocity. Hence for such cases please see this***

**Elastic collision in 2D**

**The momentum conservation**

These can be expressed as

**Kinetic energy conservation equation**

An **inelastic collision** is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).Hence some in these cases some of the energy must be transformed to deformation, sound or heat. Example: the collision between bullet and gun while a person loosely holds the gun. The collision of two balls on smooth surface, explosion of bombs, propulsion of rockets etc.

Inelastic collisions are of two types. When the objects stick together after they collide, as happens when a meteorite collides with the Earth, the collision is called **perfectly inelastic.** When the colliding objects do not stick together but some kinetic energy is transformed or transferred away, as in the case of a rubber ball colliding with a hard surface, the collision is called **inelastic** (with no modifying adverb). When the rubber ball collides with the hard surface, some of the ball’s kinetic energy is transformed when the ball is deformed while it is in contact with the surface.

**Equations for inelastic collision**

 For 1D or head collision

**Equations for perfectly inelastic collision**

**Glancing and head collision (1D and 2D collision )**

**Head-on collision** a collision in one dimension, where the objects rebound on straight line paths that coincide with the original direction of motion. This type collision occur when the center of mass of the two bodies align in the same horizontal or vertical line but not slant one.

**Glancing collision** a collision in two dimensions, where the objects rebound in the same
plane but not necessarily the same direction as the original motion. Concerning the analysis for the case of glancing collision you need to apply momentum conservation along the two dimensions since you have velocity along the two axes.

**Example (head on inelastic collision)**

A 5.2g block is fired at 2kg block which was initially at rest at the edge of frictionless table of height 2m. If the bullet embedded in to the block and does it land 1m from the ground. Then what is the initially velocity of the bullet?



Let’s start from equation of momentum since the problem involve collision

Now let’s find the velocity after collision, using equation used in projectile motion

 where

Solving this

Then

Example [from text book]

A 60 kg man standing on a stationary 40 kg boat throws a 0.2 kg ball with a velocity of 50 m/s. assuming there is no friction between the man and the boat, what is the speed of the boat after the man throws the ball?

Comment on the question

For boat to move along with the man there has to be sufficient friction between the man the and the boat otherwise the boat will not move . So let’s make little correction on that. Beside that lets assume that the friction between the boat and water to be very small. Then let’s apply conservation of linear momentum considering system comprises the collision of man plus the boat with the ball

Since there is no friction between

**Example [left for you]**

A 40.0-kg skateboarder on a 3.00-kg board is training with one 5.00-kg weight. Beginning from rest, she throws the weights horizontally, from her board. The velocity weight is relative to her after it is thrown is 7m/s to the left. Assume the board rolls without friction. (*a*) what is the velocity of her after throwing the weight? (note the relative velocity]

**Solution**

Let,

Solution

Combining the two equations

**Example (elastic collision in one dimension)**

A 4.0-kg block moving to the right at 6m/s undergoes an elastic head-on collision with a 2.0-kg block moving to the right at 3m/s .Find their final velocities.

Solution

Solution for such type’s equation

 Equation 1

From simplified equation for 1D elastic collision

 Equation 2

Combing the two equations

**Example [elastic collision in 2D, simple]**

A billiard ball having a mass of 4kg was moving at 4m/s to the right before it collides with another stationary billiard ball having a mass of 3kg. After the collision the first ball with the speed of 2m/s . Find the direction of the first ball, the velocity of the second ball

Solution

Solution

Using conservation momentum

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 Equation 1

 Equation 1

Now let’s use energy equation

Solving this

Now let’s combine the two equations, for easy of analysis let’s square the two equations and add them

Note that

Then

Solving this

Using equation 1

Solving this ,

Now lets find the actual value of using equation 2

Then

**Example 2**

A spherical clay of mass M , which was initially moving with the velocity of 3m/s to at 37 above the x axis and collides with a lump of spherical sand of mass 2M . The sand was initially moving with the speed with 2m/s at 120 above the x axis. After the collision the clay moves with the speed of 2.4m/s . Then calculate the velocity of the sand and the direction at which the clay moves after collision

Solution

Now let’s apply the momentum equation

 note

 Equation 1

 Equation 2

Now let’s now apply the second equation of elastic collision

Now let’s apply the energy for the conservation of KE

Now let’s use the first two equations to find the direction of the clay after collision

 equation 1

 equation 1

Let’s square both sides and add the two equations

Adding the two equation

Note that ,

Now squaring both sides we will come up with

Solving this

Then Now let’s use equation 1or 2 to find the angle

For taking alone

**Example [In elastic collision in 1D]**

In Fig below block 1 (mass 2.0 kg) is moving rightward at 10 m/s and block 2 (mass 5.0 kg) is moving rightward at 3.0 m/s. The surface is frictionless, and a spring with a spring constant of1120 N/m is fixed to block 2.When the blocks collide, the compression of the spring is maximum at the instant the blocks have the same velocity. Find the maximum compression (hint the

