St John Baptist De La Salle Catholic School, Addis Ababa

Grade 10 Physics Take-Home Examination Solutions
3rd Quarter

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> Notes, and use of other aids is allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work. **Cheating or indications of cheating and similar answers will be punished accordingly**.

Information

- The Take-Home Exam is due on **Tuesday**, **April 19**.
- You should Work on it **individually** and consult me if you have any questions. As I have reiterated multiple times, cheating will have a serious consequence.
- For purposes of neatness and simplicity of grading, you should do the assignment on an **A-4 paper**.

Name:______________________________ Roll Number:______ Section:______

1. (2 points) In a circuit, we are using conducting wires made from Manganese.

   (i) If we assume there are 3 free electrons per an atom of manganese, what is its electron density?

   (ii) How much current flows through a cylindrical manganese wire of volume 27 cm^3, length 3cm if the circuit is switched on for 5 seconds?

   Solution: To find the electron density of an element, we should know its number of electrons. Thus, the key part of understanding the question is how to find the number of electrons, and that is easy to find because the number of free electrons **per** atom has been given.

$$n = \frac{n_e}{v}$$

$n = \dfrac{3e^- \times n_a}{v}$, because we have 3 electrons per atom and n_a is number of atoms.

$$n = \frac{3e^- \times n_{mol} \times N_A}{v}$$
$$n = 3e^- \times \frac{m}{M} \times \frac{N_A}{v}$$
$$n = 3e^- \times \frac{m}{v} \times \frac{N_A}{M}$$
$$n = \frac{3e^- \times \rho \times N_A}{M}$$

Thus, we can use the expression above to find the electron density of Manganese that has 3 free electrons per atom.

$$n = \frac{3e^- \times 3.7g/cm^3 \times 6.022 \times 10^{23}/mol}{54.94g/mol}$$

$$n = 1.2167 \times 10^{23} e^-/cm^3$$
$$n = 1.2167 \times 10^{26} e^-/m^3$$

Once we have found the electron density, it is straight forward to find the current.

$$I = nAe^-v, \; v \textbf{ here indicates speed, not volume.}$$
$$I = n(\frac{V}{l})e^-\frac{l}{t}$$
$$I = \frac{nVe^-}{t}, \; \text{V} \textbf{ here indicates volume, not speed}$$
$$I = \frac{1.2167 \times 10^{23} e^-/cm^3 \times 27cm^3 \times 1.6 \times 10^{-19} C/e^-}{5s}$$
$$\text{I} = 105122.9 \text{ A}$$

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2. (3 points) Assume that we have an RC circuit(a circuit consisting of both a resistor and a capacitor). In the circuit, we use a capacitor of capacitance 65μF and a resistor of resistance 10KΩ.

   (i) If the source of potential difference in the circuit is a dry cell with a voltage of 12V, find how long it takes for the potential difference across the plates of the capacitor to be 7V.

   (ii) At the instant that the capacitor has a potential difference of 7V, what is the current through the circuit? In terms of percentage, by how much has the amount of current changed to 7V from the initial amount?

(iii) Let's assume that the capacitor is fully charged now and the potential difference across its plates is 12V. If we disconnect the dry cell from the circuit, the capacitor will start to act like a battery and supply its stored charge to the circuit. How much voltage would be dissipated when τ amount of time has passed?

Solution:

(i)

When a capacitor charges, we know that its voltage on the plates increase with increase in time as in the expression below:

$$V(t) = E(1 - e^{\frac{t}{\tau}})$$

First, to simplify a lot of work, let us calculate the time constant, τ

$$\tau = R \times C$$

$$\tau = 1.0 \times 10^4 \Omega \times 6.5 \times 10^{-5} F$$

$$\tau = 6.5s$$

We know that the E = 12v and here in this case, V=7V. So, let's do the math.

$$7 = 12(1 - e^{t/\tau})$$

$$\frac{7}{12} = 1 - e^{-t/\tau}$$

$$\frac{7}{12} - 1 = -e^{-t/\tau}$$

$$\frac{-5}{12} = -e^{-t/\tau}$$

Now, take the natural logarithm of each side.

$$ln(\frac{5}{12}) = ln(e^{-t/\tau})$$

$$ln(\frac{5}{12}) = \frac{-t}{\tau}$$

$$t = -\tau \times ln(\frac{5}{12})$$

$$t = 5.69s$$

(ii)

The current through the circuit initially is:

$$I_{in} = \frac{E}{R}$$

$$I_{in} = \frac{12V}{10K\Omega}$$
$$I_{in} = 1.2 \times 10^{-3} A$$

After the voltage in the plates of the capacitors increases to 7V, the current will have decreased by an amount given by:

$$I = I_{in} e^{-t/\tau}$$

We have calculated the time it takes for the potential difference across the plates of the capacitor to reach 7V and it is 5.69s. Thus,

$$I = 1.2 \times 10^{-3} A \times e^{\frac{-5.69s}{6.5s}}$$

$$I = 1.2 \times 10^{-3} A \times (0.4167)$$
$$I = 5.00 \times 10^{-4} A$$

The current has decreased by the following amount:

$$\text{increase } \% = \frac{I}{I_{in}} = \frac{5.00 \times 10^{-}4}{1.2 \times 10^{-3}} = 0.4167$$

Thus, the current has decreased by 41.67%.

Discussion Point: Why can't we just use the 7V we have to calculate the current through the circuit?

(iii)

When a capacitor discharges, it is defined with a function the initial discharge voltage as follows:

$$V(t) = V_{in} e^{\frac{-t}{\tau}}$$

We have been told to find the voltage at the instant the time is RC, or $t = \tau$. Thus,

$$V(\tau) = 12V e^{\frac{-\tau}{tau}}$$

$$V(\tau) = 12V e^{-1}$$
$$V(\tau) = 4.14V$$

Thus, we see that the voltage drops rapidly to 4.14V as the capacitor discharges through τ.

3. (2 points) A 1.5V battery has an internal resistance of 0.3Ω. A load of variable resistance is connected across the battery and adjusted to have resistance 4 times resistance to that of the internal resistance of the battery.

   (i) Find the total power dissipated in the circuit.

(ii) Find the current through the circuit.

(iii) Find the potential difference across the terminals of the battery.

Solution:

(i)

To find the total power, we first need to calculate the total resistance in the circuit. We have been given the value of the internal resistance(r) and told that the load resistance(R) is four times as larges as r. Thus,

$$R = 4r$$

$$R = 4(0.3\Omega)$$

$$R = 1.2\Omega$$

Thus, the total resistance in the circuit is r+R.

$$R_t = r + R$$

$$R = 0.3\Omega + 1.2\Omega = 1.5\Omega$$

The total power dissipated in the circuit is given by:

$$P = \frac{E^2}{R}$$

$$P = \frac{(1.5V)^2}{1.5\Omega}$$

$$P = 1.5W$$

(ii)

To find the current, recall that:

$$E = I(r + R)$$

$$I = \frac{E}{r + R}$$

$$I = \frac{1.5V}{1.5\Omega}$$

$$I = 1A$$

(iii)

The terminal voltage is given by: V = IR, thus

$$V = 1A \times 1.2\Omega$$

V= 1.2V Thus, if we had measured the potential difference along the terminals of the battery(terminal voltage), the measurement we would get is 1.2V not the 1.5V written in the battery. Why? We just saw precisely why: **internal resistance**.

4. (1 point) Explain how current density($\mathbf{J}$) is related to Ohm's Law. Give an expression for resistivity(ρ) in terms of the cross-sectional area of a conductor and the current passing through it.

Read the posted notes to attempt this question.

Solution:

In electromagnetism, we use the current density definition of Ohm's Law as it tells us what really is happening in the microscopic level. It is a function of electric field and conductivity at any given point in a material to give the current density.

$$J = \sigma E$$
$$\frac{I}{A} = \sigma \frac{V}{L}$$
$$V = \frac{IL}{\sigma A}$$
$$V = I \times \frac{L}{\sigma A}$$
$$V = I \times R, \text{ since } \frac{L}{\sigma A} = R$$

To answer the second part of the question;

$$J = \sigma E$$
$$J = \frac{1}{\rho} E$$
$$\rho = \frac{E}{J}$$

5. (3 points) A 5.0μF parallel-plate capacitor is connected to a constant voltage source. If the distance between the plates of this capacitor is 4mm and the capacitor holds a charge of 13.6μC.

   (i) What is the strength of the electric field between the plates of this capacitor?

   (ii) If an electron was to be placed in between the plates of the capacitor, how much force would it experience?

Solution:
(i)

To find the strength of the electric field, we first need to find the potential difference between the plates of the capacitor.

$$Q = CV$$

$$V = \frac{Q}{C}$$
$$V = \frac{13.6\mu C}{5\mu F}$$
$$V = 2.72V$$

Now, we can calculate the electric field assuming the separation of the plates as the distance over which the electric field acts.

$$V = Ed$$
$$E = \frac{V}{d}$$
$$E = \frac{2.72v}{4 \times 10{-}3m}$$
$$E = 680N/C$$

(ii)

If an electron was placed in between the plates of the capacitor, it would an experience a force given by:

$$F = qE$$
$$F = e^{-}E$$
$$F = 1.6 \times 10^{-19}C \times 680N/C$$
$$F = 1.088 \times 10^{-16}\ \mathrm{N}$$